TRINITY COLLEGE

Mathematics Specialist Unit 1&2
Test 3 2018

Calculator Assumed
Proof

STUDENT'S NAME ______________________________

DATE: Thursday 17 May **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (1 mark)

Consider the following statement:

All prime numbers when squared are odd.

Provide a counter example that shows this statement is false.

$2^2 = 4$

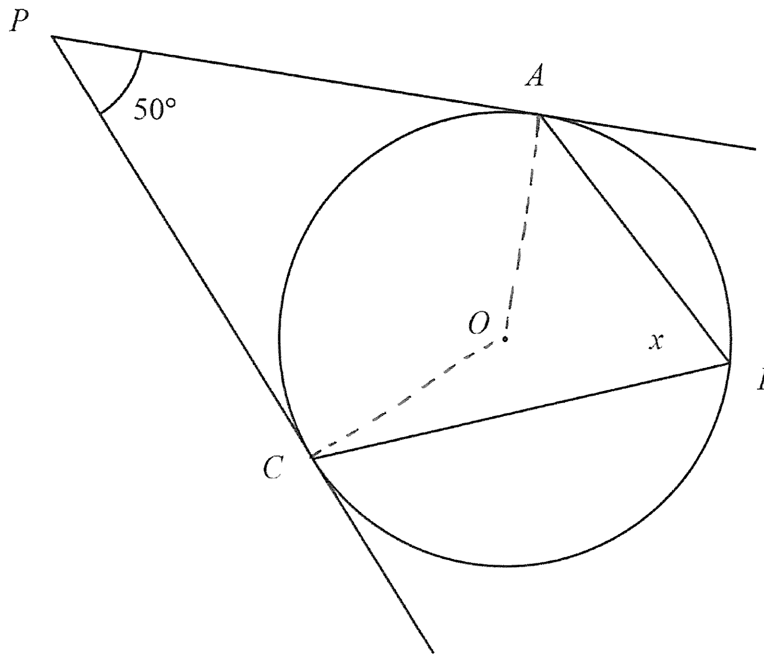
2 is a prime number

4 is even

2. (8 marks)

Determine, with reasons, that value of each unknown

(a) [4]



$\angle PAO = 90°$ (tangent)

$\angle PCO = 90°$ "

$\Rightarrow \angle AOC = 180 - 50°$

$= 130°$

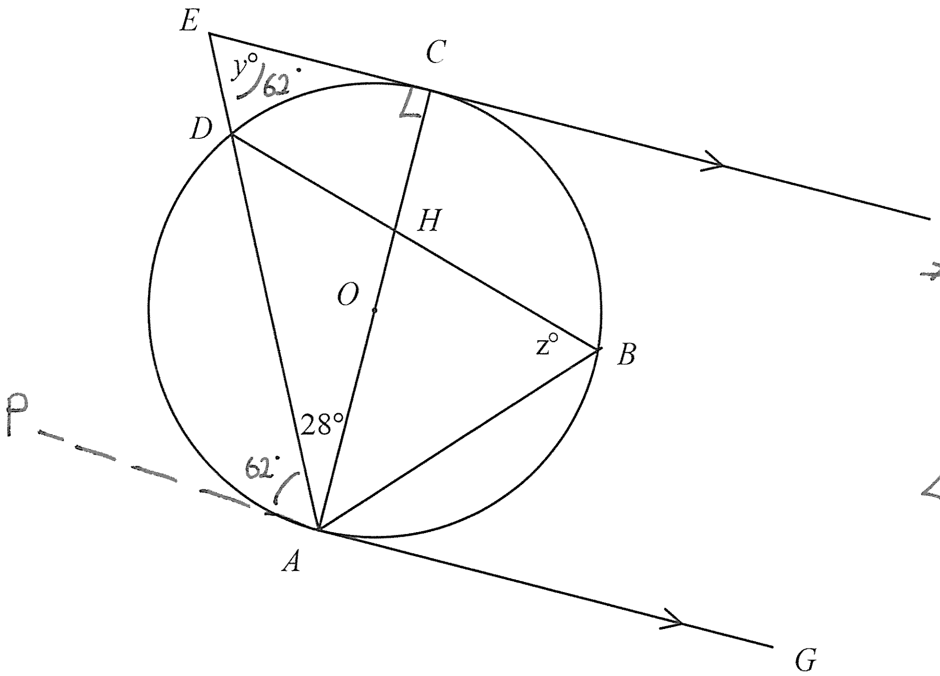
(angles quadrilateral)

$\Rightarrow \angle ABC = 65°$

(angle at circumference ½ angle at centre)

$\therefore x = 65°$

(b) [4]



$\angle ECA = 90°$ (tangent)

$\Rightarrow y = 90 - 28$

$= 62°$ (angles $\triangle$)

$\angle PAE = 90 - 28$

$= 62°$ (tangent)

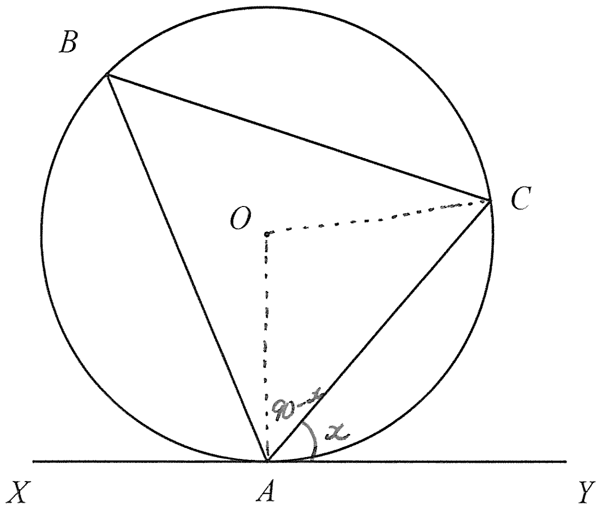
$\Rightarrow z = 62°$

(Alternate segment)

3. (5 marks)

Prove the Alternate Segment Theorem

i.e. for the circle below, centre O, prove $\angle CAY = \angle ABC$



Let $\angle YAC = x$

$\overline{OA}$ and $\overline{OC}$ are both radii

$\angle OAY = 90^\circ$ (tangent)

$\Rightarrow \angle OAC = 90^\circ - x$

$\Rightarrow \angle OCA = 90 - x$ (issosceles $\triangle$)

$\Rightarrow \angle COA = 180 - 2(90 - x)$

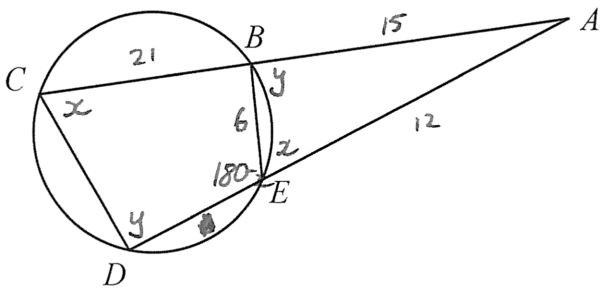
$= 2x$ (angles $\triangle$)

$\Rightarrow \angle ABC = x$ (angle at circumference ½ angle centre)

$\therefore \angle CAY = \angle ABC$ QED

4. (6 marks)

Two sides of the cyclic quadrilateral $BCDE$ are extended to meet at A, as shown in the diagram.



(a) Prove that triangles ADC and ABE are similar. [3]

Let $\angle BCD = x$

$\Rightarrow \angle BED = 180 - x$ (opp cyclic quad)

$\Rightarrow \angle BEA = x$ (supplement)

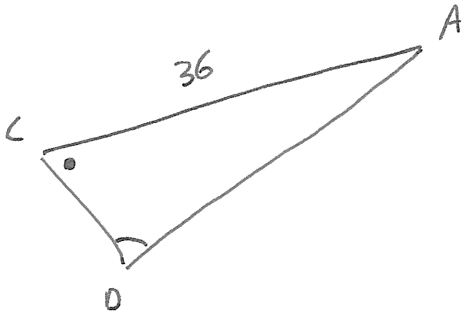
Similary, let $\angle CDE = y$

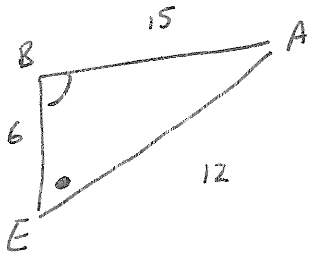
$\Rightarrow EBA = y$ ״

$\therefore \triangle ADC \sim \triangle ABE$ (AA)

(b) If $AB = 15$, $BC = 21$, $AE = 12$ and $BE = 6\,cm$, determine the lengths of DE and CD.

[3]





$\frac{DA}{15} = \frac{36}{12}$

$\Rightarrow DA = 45$

$\therefore DE = 45 - 12$

$= 33$ cm

$\frac{CD}{6} = \frac{36}{12}$

$\Rightarrow CD = 18$ cm

TRINITY COLLEGE

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STUDENT'S NAME Solutions

DATE: Thursday 17 May **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (6 marks)

Write each of the following mathematical statement in words:

(a) *for all* x $\exists y$ such that $y < x$ [2]

"for all x there exist a y such that y is less than x"

(b) $x^3 = y^3 \Rightarrow x = y$ [1]

"If x^3 equals y^3, then this implies x equals y"

(c) for the above statement in part (b);

(i) Write down the converse of this statement and state whether it is true or false, and if it is false, provide a counter-example. [2]

"If x equals y, then this implies x^3 equals y^3"

This is true

(ii) Amend the statement in part (b) using an equivalence statement. [1]

$x^3 = y^3 \Longleftrightarrow x = y$

6. (7 marks)

Consider the following statement:

If you draw any nine playing cards from a standard deck, then you will have at least three cards all of the same suit.

(a) Prove this statement. [3]

Let the suits be the pigeon holes and the cards be the pigeons

∴ by the PHP there must be at least one suit that has at least 3 cards.

∴ the above statement is correct

(b) Write down the contrapositive of this statement and state whether it is true or false, and if it is false, provide a counter-example. [2]

"If you do not have at least 3 cards of the same suit, then you do not have (at least) 9 cards"

True

(c) Write down the inverse of this statement and state whether it is true or false, and if it is false, provide a counter-example. [2]

"If you don't have 9 playing cards, then you don't have at least 3 cards of the same suit"

False, if you only have 52 cards you have at least 3 cards of <u>all</u> suits!

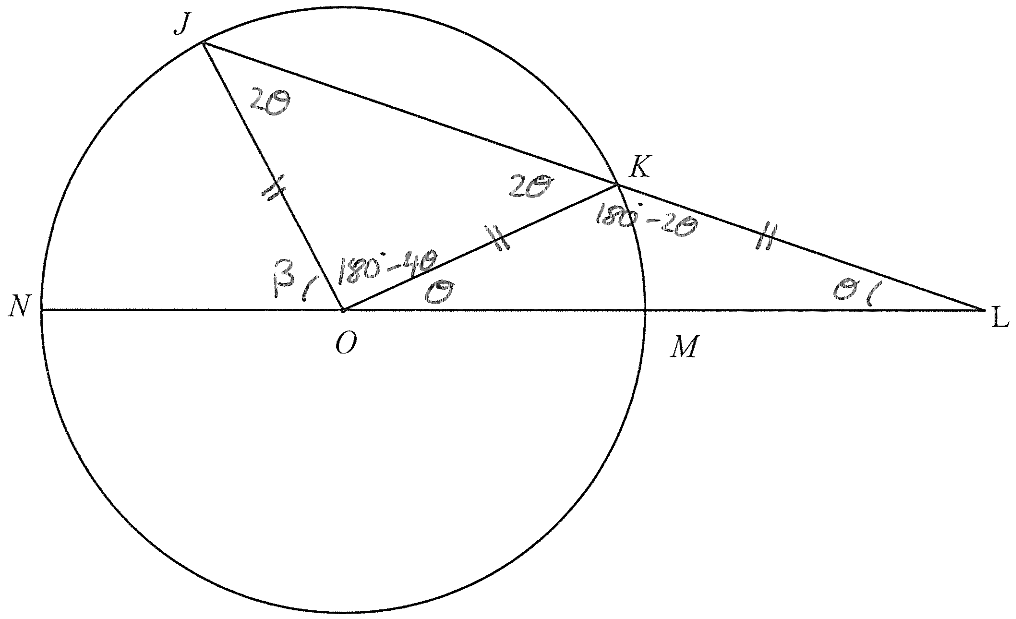
7. (6 marks)

The points J, K, M and N are points on the circumference of the circle centre O, shown below.

Let $\angle JON = \beta$ and $\angle KLM = \theta$

The length KL is equal to the radius of the circle.

Prove that $\beta = 3\theta$



$\angle KOL = \theta$ (issosceles Δ)

$\angle OKL = 180^\circ - 2\theta$ (angle sum Δ)

$\angle OKJ = 2\theta$ (straight line)

$\angle OJK = 2\theta$ (issosceles Δ)

$\angle JOK = 180^\circ - 4\theta$ (angles Δ)

$\angle JON = 180 - (180 - 4\theta) - \theta$ (straight line)

$= 3\theta$

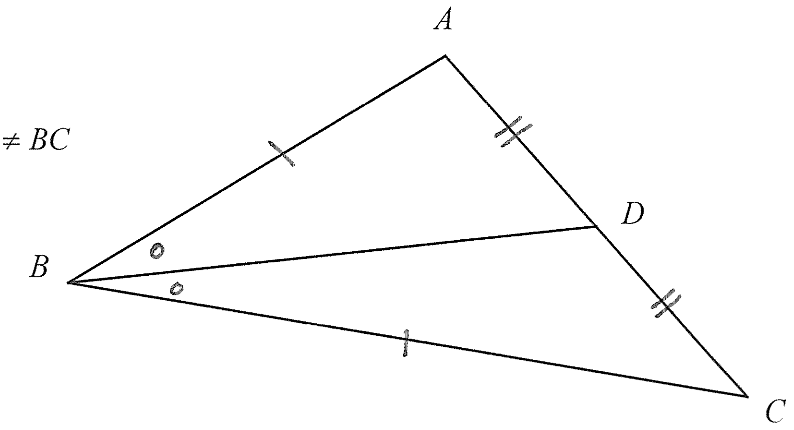
$\therefore \beta = 3\theta$ QED

8. (4 marks)

Consider the diagram with the following information:

BD bisects $\angle ABC$
$\angle ADB$ is acute

Prove, by contraction, that $AB \neq BC$



Assume $AB = BC$

BD bisects $\angle ABC \Rightarrow AD = DC$

and AD is common

and $AB = BC$

$\therefore \triangle BAD \cong \triangle BCD$ (SSS)

$\therefore \angle ADB = \angle CDB$ (congruent $\triangle$)

$\Rightarrow \angle ADB + \angle CDB = 180$

$\Rightarrow \angle ADB = 90°$

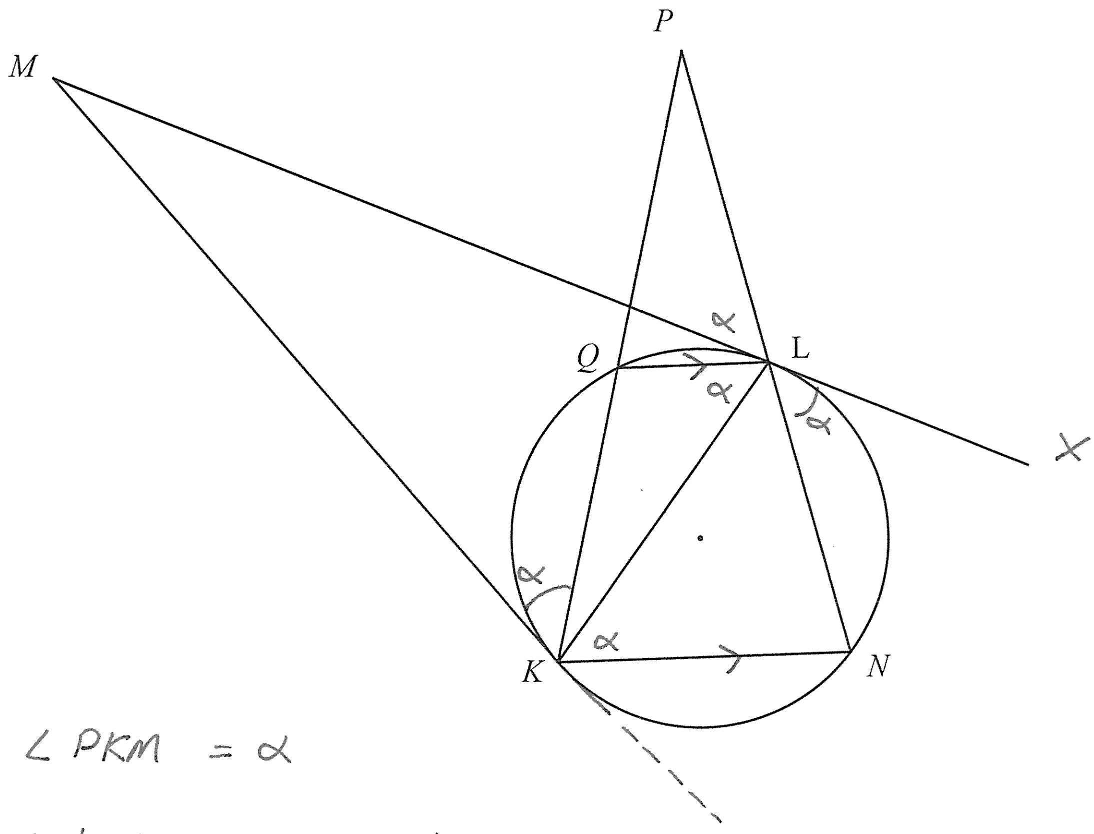
But $\angle ADB$ is acute. This is a contradiction

$\therefore$ original assumption is false

$\therefore AB \neq BC$

9. (7 marks)

If MK and ML are tangents to the circle and $KN \parallel QL$, prove that $MKLP$ is a cyclic quadrilateral.



Let $\angle PKM = \alpha$

$\Rightarrow \angle KLQ = \alpha$ (Alt segment)

$\Rightarrow \angle LKN = \alpha$ (Alt angle // lines)

$\Rightarrow \angle XLN = \alpha$ (Alt segment)

$\Rightarrow \angle PLM = \alpha$ (Vert opp)

$\therefore \angle PKM = \angle PLM$

$\Rightarrow$ Both L and K are on an arc subtended by P and M

$\Rightarrow$ L, K, P and M are all on the same arc (circle)

$\therefore$ MKLP is a cyclic quadrilateral